İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

RECONSTRUCTION OF CRACKS LOCATED IN A LAYERED MEDIA

M.Sc. Thesis by
Kemal MRKONJA

Department : Electronics and Telecommunication Engineering
Programme : Telecommunication Engineering

Thesis Supervisor: Prof. Dr. İbrahim AKDUMAN

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Kemal MRKONJA
(504071339)

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Supervisor : **Prof. Dr. İbrahim AKDUMAN(ITU)**
Member of the Examining Committee : **Prof. Dr. Tayfun GÜNEL(ITU)**
Asst. Prof. Dr.HülyaŞAHİNTÜRK(YTU)

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TABAKALI YÜZEY ALTINDA KONUMLANDIRILMIŞ ÇATLAKLARIN TESPİT EDİLMESİ

YÜKSEK LİSANS TEZİ

Kemal MRKONJA
(504071339)

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Dığer Jürı Üyelerı : Prof. Dr. Tayfun GÜNEL(ITÜ)
Asst. Prof. Dr. Hülya ŞAHİNTÜRK(YTÜ)

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ABBREVIATIONS

MoM : Method of Moments
ACM : An Analytic Continuation Method

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RECONSTRUCTION OF CRACKS LOCATED IN LAYERED MEDIA

SUMMARY

The scattering and inverse scattering problems related to crack located in a layered media are of importance due to their potential applications such as early detection of earthquakes, solidity of building constructions or large machine components, since the presence of cracks may cause undesirable consequences. Crack's early detection is of great importance and is a challenging research area for scientist and engineers of various fields. Although several research activities have been carried out, the subject still needs to be investigated to obtain more stable and accurate results.

The main aim of this thesis is to analyze both direct and inverse scattering problems related to cracks buried in a half-space. In this direction, the crack in soil, for example, is modelled in terms of perfect electric conductivity strips. In both direct and inverse scattering problems, it is assumed that the problem is two dimensional 2D and the incident wave is a plane wave. The direct scattering problem is solved by reducing the problem to the solution of an integral equation via the Green's function of the two half-space medium. The integral equation is solved by an application of the method of moments (MoM). The end point contributions of the cracks are ignored by giving a small thickness to the cracks.

The aim of the inverse scattering problem related to a crack is to determine its shape and location through the measurement of the scattered field on a certain line in the half-space not containing the crack. By using the fact that the total electric field vanishes on the crack surface, the reconstruction is achieved. The methods are tested by considering some illustrative examples.

TABAKALI YÜZEY ALTINDA GÖMÜLÜ ÇATLAKLARIN TESPİT EDİLMESİ

ÖZET

Tabakalı bir ortamdaki çatlaklara ilişkin düz ve ters saçılma problemleri, bulunan çatlakların arzu edilmeyen sonuçlar doğurması sebebiyle depremlerin erken tespiti, binaların veya büyük makina parçalarının sağlamlığı gibi potansiyel uygulamalar açısından büyük bir önem taşımaktadır. Çok çeşitli alanlarda çalışan bilim adamları ve mühendisler için çatlakların erken tespiti önemli ve zorlayıcı bir araştırma alanıdır. Çeşitli araştırmalar yapılmış olmasına rağmen bu konu, daha kararlı ve kesin sonuçlar elde edilebilmesi için yeni araştırmalara ihtiyaç duymaktadır.

Bu tezin asıl amacı, yarı uzaya gömülü çatlaklara ilişkin hem düz hem de ters saçılma problemlerinin analizini yapmaktır. Bunun için örneğin toprak içindeki çatlak, mükemmel elektrik iletken bir şerit olarak modellenmiştir. Hem düz hem de ters saçılma problemlerinde, problemin geometrisinin 2 boyutlu, gelen dalganın ise düzlemsel dalga olduğu varsayılmıştır. Düz problem, çözümün iki yarı uzaya ilişkin Green fonksiyonu yardımıyla bir integral denkleme indirgenmesiyle çözülmüştür. İntegral denklem, moment metodu'nun (MoM) bir uygulaması olarak çözülmüştür. Çatlakların uç noktalarının katkıları, çatlaklara çok küçük bir kalınlık verilerek ihmal edilmiştir.

Çatlağa ilişkin ters saçılma probleminin amacı, çatlağı içermeyen yarı uzayda bulunan bir çizgi üzerindeki saçılan alan ölçümleri kullanılarak çatlağın yerinin ve şeklinin bulunmasıdır. Çatlağın yüzeyi üzerinde toplam elektrik alanın yok olması gerçeğine dayanarak rekonstrüksiyona ulaşılmıştır. Kullanılan metotlar, görsel örnekler dikkate alınarak test edilmiştir.

1. INTRODUCTION

The basic ideas of detection and identification of objects are constructed on the idea and phenomena of the wave propagation and wave scattering. Electromagnetic waves can propagate through various media, be reflected or scattered and received or detected by sensors. Different properties of different objects give possibilities to be detected by sensors. The human body has two examples of such sensors, ears and eyes. The eyes sense electromagnetic waves in the visible part of the electromagnetic spectrum, and the ears sense acoustic waves. Phenomenon of electromagnetic interaction with obstacles, such as reflection, refraction and scattering was first studied for case of optical spectrum, namely visible portion of electromagnetic spectrum. However, human's need for observations of the different obstacles has brought new challenges for scientists and engineers to move the spectrum of their researches out of the visible portion, which leads to improvement in medical imaging, remote sensing, distance-testing, microelectronics, geophysics, mine detection, communication industry and many others.

Although detection of obstacles in free space has been well investigated, detection of the obscured and hidden objects has been and still is a challenging area of interest. Due to the interaction of electromagnetic waves with the surrounding medium, research of obscured targets still remains as an emergent field, which requires precise and accurate investigation. In most of the problems object is surrounded by a complex medium which makes the investigation more and more challenging.

Scattering has become a popular interest of research in electromagnetic problems. Scattering problems can be divided into two main classes, direct and inverse scattering problems. Direct scattering problems investigate the behaviour of the electromagnetic waves in interaction with obstacles whose electrical properties are known, in a medium with known electrical properties, illuminated by a known source. On the other hand, inverse scattering problems focus on reconstructing the location and the electrical properties of an inaccessible target in a medium illuminated by a known source. The main difficulty of solving an inverse scattering problem is that the boundary conditions on the target are assumed to be known. However, this approach is clearly insufficient in real life applications; thus, different boundary conditions are defined to overcome this disagreement between practical and theoretical applications. There is a great deal of publications and books on both subjects [1, 2].

1.1 Inverse Scattering Problem Related to Buried Objects and Importance of the Crack Detection

The importance of investigation of the problems related to buried objects is still a very attractive subject due to the fact that many areas need qualitative applications, and above all, accurate results. Some of the application areas are detection and location of the mines, non-destructive testing, crack detection in earthquake zone, underground tunnel detection, pipelines, medical imaging etc. In electromagnetic theory those applications are recognized as 'inverse scattering problem' which have great contribution to many practical applications. A property of the objects to be reconstructed mainly determines solution approaches of the related inverse scattering problem. If object to be reconstructed is not penetrable such as perfect electric conductor (PEC) for the electromagnetic case, the problem becomes a shape reconstruction problem. Contrariwise, if the object to be reconstructed is penetrable such as dielectric objects in electromagnetic sense, the solution approach should be established to reconstruct not only the shape but also the electrical parameters of the object [3].

In this thesis, shape reconstruction problem will be considered, that is to say impenetrable perfect electric conductor object shaped as crack will be investigated. Up to now several analytical and numerical methods have been developed for investigation of the buried objects [4,5-7], as well as investigation of the cracks [8,9]. The very first researches on subject of inverse scattering problems were in late 60's and 70's [10,11]. From that time until now many methods have been presented, and it is convenient to mention some of them such as physical optics theory, equivalent source method, Newton-Kantorovich method, probe method, linear sampling method, factorization method, decomposition method [12-18] etc. Roughly the methods can be sorted as iterative, probe and decomposition or analytical continuation method (ACM). In this thesis analytical continuation method, in which a linear operator is used to reconstruct the scattered field in the proximity of the obstacle with usage of the a priori measured far field, will be handled. Applying boundary conditions, the shape reconstruction problem is formulated as a minimization of the non-linear equation of the shape.

1.2 Aim of the Thesis

The aim of this study is to discuss the model of the two layered medium and its effect on electromagnetic wave scattered from the buried perfectly conducting object, crack namely. First of all, direct scattering model of two half-spaces will be discussed. This model is represented by the Green's function of two half-space media, which reduces the problem to the solution of the Fredholm integral equation of the first kind, which is then solved by using the well-known Method of Weighted Residuals or with more common name Method of Moments (MoM) [19-21]. After solution of the direct scattering problem we are provided with the scattered field. Then this limited far field data information is used for reconstruction of the inaccessible object. The method used for the reconstruction of the buried object is based on the analytical continuation method (ACM) [22] described in section 3.2. Afterwards, several numerical examples are shown in addition to validate analytical continuation method. Final conclusions and future work recommendations are given in the last part of the thesis.

2. DIRECT SCATTERING PROBLEM RELATED TO CRACKS IN A HALF-SPACE

In this section the direct scattering problem related to buried crack will be considered and a numerical method based on method of moments (MoM) will be presented.

2.1 Problem Description

Direct scattering configuration for case of two-half spaces separated by planar interface is considered. In Figure 2.1 the geometry of the direct problem is illustrated, where the whole space consists of two half spaces, two different medium namely, and is separated by the planar interface Γ_0. The electromagnetic parameters of the upper and lower half-spaces are $\varepsilon_1, \mu_0, \sigma_1 \approx 0$ and $\varepsilon_2, \mu_0, \sigma_2$, respectively. The strip can be considered as a model for a crack located in the lower half-space $x_2 < 0$. Then, the scattering problem considered here is to determine the effect of L on the propagation of electromagnetic waves excited in the upper half-space $x_2 > 0$, more precisely, to obtain the scattered field from the crack L. To this aim, the half-space $x_2 < 0$ is illuminated by a time-harmonic plane wave, whose electric-field vector is always parallel to the Ox_3 axis, namely,

$$\vec{E}_i = (0, 0, u_i(x_1, x_2)) \tag{2.1}$$

$$u_i(x_1, x_2) = e^{-ik_1(x_1 \cos\phi_0 + x_2 \sin\phi_0)} \tag{2.2}$$

where $\phi_0 \in (0, \pi)$ is the incident angle, and k_1 is the wave number of the upper half-space, which is defined as $k_1 = \sqrt{\omega^2 \varepsilon_1 \mu_0 + i\omega\sigma_1\mu_0}$. Since the problem is homogeneous in the Ox_3 direction, the total electric-field vector will also be parallel to the Ox_3*-axis* i.e., where (x_1, x_2) denotes the position vector in $\Re^2$. Thus, the problem reduces to a scalar one in terms of the total field function u(x), which satisfies Helmholtz equation.

$\Delta u + k^2(x)u = 0$ **(2.3a)**

with boundary conditions

u is continuous on Γ_0 **(2.3b)**

$\dfrac{\partial u}{\partial x_2}$ *is continuous on* Γ_0 **(2.3c)**

$u = 0$ *on* L **(2.3d)**

under the appropriate radiation condition. In order to formulate the problem in an appropriate way $u(x)$ is firstly separated as $u(x) = u_0(x) + u_s(x)$. The field $u_0(x)$ is the total field in the absence of the body L, while $u_s(x)$ denotes the contribution of L, which is the scattered field due to L.

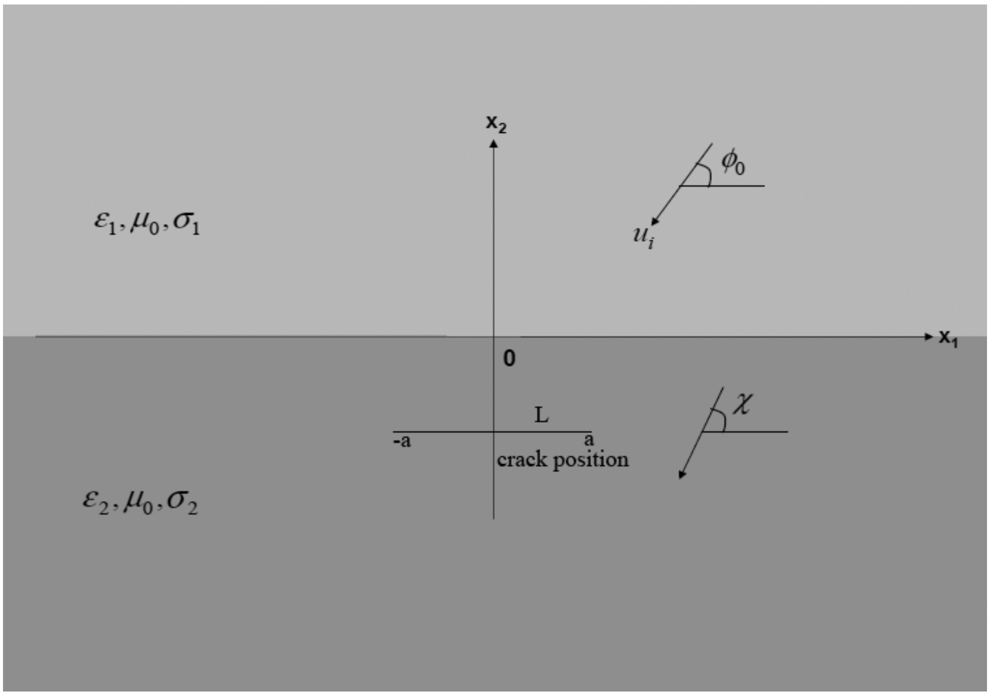

Figure 2.1: Geometry of the problem for direct scattering

The field $u_0(x)$ is the solution of the problem,

$\Delta u_0 + k^2(x_2)u_0 = 0$ **(2.4a)**

u_0 *is continuous on* Γ_0 **(2.4b)**

$\dfrac{\partial u_0}{\partial x_2}$ *is continuous on* Γ_0. **(2.4c)**

The solution to this particular problem is very straightforward and can be easily obtained from any subject related textbook.

Then one has,

$$u_0(x) = \begin{cases} u_i(x) + \mathrm{R}\, e^{-ik_1(x_1\cos\phi_0 - x_2\cos\phi_0)}, & x_2 > 0 \\ Te^{-ik_2(x_1\cos\chi - x_2\cos\chi)} \qquad\qquad , & x_2 < 0 \end{cases} \tag{2.5}$$

where χ is the transmission angle given by Snell's law,

$$k_1 \cos\phi_0 = k_2 \cos\chi \tag{2.6}$$

while R and T represent reflection and transmission coefficients, respectively, of the plane x_2=0, and

$$R = \frac{k_1 \sin\phi_0 - k_2 \sin\chi}{k_1 \sin\phi_0 + k_2 \sin\chi} \tag{2.7a}$$

$$T = \frac{2k_1 \sin\phi_0}{k_1 \sin\phi_0 + k_2 \sin\chi}. \tag{2.7b}$$

We now consider the scattered field $u_s(x)$ due to the crack L. It should be noted that the result of such problems are of interest in the areas of geology, seismology, remote sensing etc. and constitutes a difficult problem in electromagnetic theory. The $u_s(x)$ satisfies the reduced wave equation,

$$\Delta u_s + k^2(x_2)u_s = 0 \tag{2.8}$$

and the boundary conditions

$$u_s(x) \textit{ is continuous } \Gamma_0, \tag{2.8a}$$

$$\frac{\partial u_s(x)}{\partial x_2} \textit{ is continuous on } \Gamma_0 \tag{2.8b}$$

$$u_s(\mathrm{x}) = -u_0(\mathrm{x}) \textit{ on } L \tag{2.8c}$$

with the appropriate radiation condition. A convenient way to solve the scattering problem given by equation 2.8 is to reduce it to the solution of an integral equation. It can be achieved through the Green's function of the background medium. Let $G(\mathrm{x};\mathrm{y})$ denotes the Green's function, in this case through the Green's theorem one can write

$$u_s(\mathrm{x}) = j\omega\mu_0 \int_L G(\mathrm{x};\mathrm{y})J(\mathrm{y})dy \qquad \textbf{(2.9)}$$

where $J(\mathrm{y})$ denotes the surface currents induces on L. By the use of the boundary conditions given in equation 2.8a-2.8c we get

$$u_0(\mathrm{x}) = j\omega\mu_0 \int_L G(\mathrm{x};\mathrm{y})J(\mathrm{y})dy \;\; on \;\; L \qquad \textbf{(2.10)}$$

which constitutes an Fredholm integral equation of the first kind for the solution of unknown current density $J(\mathrm{y})$.

Before applying any solution procedure for the integral equation, one has to consider end contributions of the J to the scattered field. In the following we will apply the method of moments to the solution equation given in 2.10. In such a case we will assume that L has a thickness or more precisely that is constructed as a two dimensional object, and the thickness will be taken very small.

Let us first expand the current density J via set of basis functions ϕ_i, i=1,...N having supports on L that spans a polynomial space of order p defined on L, namely,

$$J(y) = \sum_i^N I_i \phi_i(y). \qquad \textbf{(2.11)}$$

It is necessary to mention that the quality of the interpolation is based on the choice of ϕ_i and N. Substituting (2.11), (2.10) becomes,

$$u_0(\mathrm{y}) = j\omega\mu_0 \sum_i^N I_i \int_{Curve} G(\mathrm{y};\mathrm{y}')\phi_i(\mathrm{y}')dy \qquad \textbf{(2.12)}$$

It can obviously be seen in the expression (2.12) that I_i are the unknown coefficients that need to be solved. In order to formulate a linear system of equations, N-moments of the field operator will be computed. The next step for this aim is to introduce a set of test function space with support on the surface of the crack that spans the polynomial space $\{\psi_i\}$ of order d defined on the crack curve. Then, the inner product of the (2.12) is calculated as follows,

$$\langle \psi_j, u_0(\mathrm{y})\rangle = j\omega\mu_0 \sum_i^N I_i \left\langle \psi_j, \int_{Curve} G(\mathrm{y};\mathrm{y}')\phi_i(\mathrm{y}')dy \right\rangle \qquad \textbf{(2.13)}$$

where,

$$\langle f, g \rangle = \int_{Curve} f(y')g(y')dy \qquad \textbf{(2.14)}$$

The expression (2.14) mentioned above can be written as a linear system of equations as [21]:

$$\left[U_j\right] = \left[I_i\right]\left[Z_{ij}\right] \qquad \textbf{(2.15)}$$

Where

$$U_j = \left\langle \psi_j, u_0(\mathrm{y}) \right\rangle \qquad \textbf{(2.15a)}$$

and

$$Z_{ij} = \left\langle \psi_j, \int_{Curve} G(\mathrm{y};\mathrm{y}')\phi_i(\mathrm{y}')dy \right\rangle \qquad \textbf{(2.15b)}$$

Expression Z_{ij} denotes an impedance matrix and assuming that there are N testing functions this is a $N \times N$ dimensional matrix. Along with this and some more matrix operations previously unknown current distribution over the crack surface is obtained. It is important to mention that the basis functions are assumed to be pulse basis functions, and the testing functions are chosen as point test function i.e. delta-Dirac functions.

2.2 Green's Function of Two-Layered Medium

The Green's function of the two-half spaces separated by a planar interface satisfies following equation [24],

$$\Delta G(\mathrm{x};\mathrm{y}) + k^2(x_2)G(\mathrm{x};\mathrm{y}) = -\delta(\mathrm{x}-\mathrm{y}) \qquad \textbf{(2.16)}$$

where δ is the Dirac's delta distribution and $y \in \Re^2$ is an arbitrary point. To solve the above Green's function, we first consider its Fourier transform with respect to x_1 , namely,

$$\hat{G}(\upsilon, x_2; \mathrm{y}) = \int_{-\infty}^{\infty} G(\mathrm{x};\mathrm{y})e^{-i\upsilon x_1}dx_1 \qquad \textbf{(2.17)}$$

Applying (2.17) to (2.16) and considering the boundary conditions at $x_2 = 0$ given in (2.19), and the radiation condition given in (2.20), it yields the following expression for the problem of solving the Green's function G ,

$$\frac{d^2}{d\ x_2^{\ 2}}\hat{G} - \left[\upsilon^2 - k_j^{\ 2}\right]\hat{G} = -e^{-i\upsilon y_1}\delta(x_2 - y_2),\ j = 1,2\ ,\upsilon \in C_R \qquad \textbf{(2.18)}$$

$$\hat{G} \text{ and } \frac{\partial \hat{G}}{\partial x_2} \text{ are continuous on } x_2 = 0 \qquad \textbf{(2.19)}$$

$$\left|\hat{G}\right| \longrightarrow 0 \text{ as } |x| \longrightarrow \infty \qquad \textbf{(2.20)}$$

C_R in (2.18) stands for a horizontal straight line in the regularity strip of $\hat{G}$ in the complex υ-plane shown in Figure 2.2.

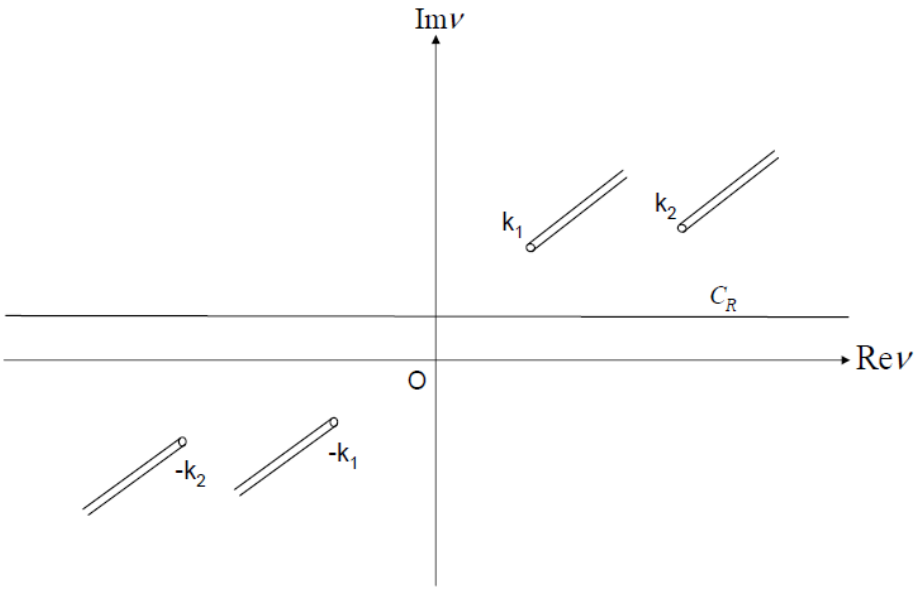

Figure 2.2: – Complex υ-plane

After some straightforward calculations and through the well-known inverse Fourier transform integral,

$$G(\mathrm{x};\mathrm{y}) = \frac{1}{2\pi}\int_{C_R} \hat{G}(v, x_2; \mathrm{y})\ e^{-ivx_1} dv \qquad \textbf{(2.21)}$$

the explicit expression of $G(\mathrm{x};\mathrm{y})$ is obtained as follows:

$$G(\mathrm{x};\mathrm{y}) = \begin{cases} \frac{i}{4}H_0^{(1)}(k_1|x-y|) + G_R^{(1)}(x;y); & \mathrm{x}_2>0,\ \mathrm{y}_2>0 \\ G_T^{(1)}(x;y); & \mathrm{x}_2<0,\ \mathrm{y}_2>0 \\ G_T^{(2)}(x;y); & \mathrm{x}_2>0,\ \mathrm{y}_2<0 \\ \frac{i}{4}H_0^{(1)}(k_2|x-y|) + G_R^{(2)}(x;y); & \mathrm{x}_2<0,\ \mathrm{y}_2<0 \end{cases} \quad \textbf{(2.22)}$$

where

$$G_R^{(1)}(x;y) = \frac{1}{2\pi}\int_{C_R} \frac{1}{2\gamma_1}\,\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\, e^{-\gamma_1(x_2+y_2)}\, e^{iv(x_1-y_1)}dv \quad \textbf{(2.23)}$$

$$G_T^{(1)}(x;y) = \frac{1}{2\pi}\int_{C_R} \frac{1}{2\gamma_1}\,\frac{2\gamma_1}{\gamma_1+\gamma_2}\, e^{-\gamma_1 y_2+\gamma_2 x_2}\, e^{iv(x_1-y_1)}dv \quad \textbf{(2.24)}$$

$$G_T^{(2)}(x;y) = \frac{1}{2\pi}\int_{C_R} \frac{1}{2\gamma_2}\,\frac{2\gamma_2}{\gamma_1+\gamma_2}\, e^{\gamma_2 y_2+\gamma_1 x_2}\, e^{iv(x_1-y_1)}dv \quad \textbf{(2.25)}$$

$$G_R^{(2)}(x;y) = \frac{1}{2\pi}\int_{C_R} \frac{1}{2\gamma_2}\,\frac{\gamma_2-\gamma_1}{\gamma_1+\gamma_2}\, e^{-\gamma_2(x_2+y_2)}\, e^{iv(x_1-y_1)}dv \quad \textbf{(2.26)}$$

while $H_0^{(1)}$ denotes zero order Hankel function of the first kind. In (2.23-2.26) the functions γ_1 and γ_2 stand for the square roots,

$$\gamma_1(v) = \sqrt{v^2-k_1^{\,2}}, \quad \gamma_2(v) = \sqrt{v^2-k_2^{\,2}} \quad \textbf{(2.27)}$$

which are defined in the complex υ-plane cut as shown in Figure 2.2 with the conditions

$$\gamma_j(0) = \text{-ik}_j,\ \ j=1,2. \quad \textbf{(2.28)}$$

From the expressions (2.23-2.26) and (2.27) it can easily be seen that $G(x;y)$ is symmetrical, i.e.:

$G(\mathrm{x};\mathrm{y}) = G(\mathrm{y};\mathrm{x})$ and has the property

$$G(\mathrm{x};\mathrm{y}) = \hat{G}(|x_1-y_1|;x_2,y_2), \qquad (\mathrm{x},\mathrm{y})\in\Re^2 \quad \textbf{(2.29)}$$

2.3 Expression of the Scattered Field

Hence, since the current on the crack and the Green's function of the two half spaces are now known, the scattered field given in (2.9) can be directly calculated by

$$u_s(\mathrm{x}) = j\omega\mu_0 \sum I_i \int_{curve} G(\mathrm{x};\mathrm{y})dy \qquad \textbf{(2.30)}$$

Simulation results of the scattered field and the reconstructed crack are shown in section 5.

3. CRACK RECONSTRUCTION

Reconstructing the location and the shape of the objects, namely cracks, has been an active research subject for a few decades [6, 9, 13]. Aim of these inverse scattering problems is to extract the information such as location, shape and composition of inaccessible objects from the measurement of the scattered field in far-field region. Versatility of the applications makes the research in this area more attractive.

3.1 Problem Setup

In Figure 3.1, the problem of interest is represented. In this representation, the inverse scattering problem is defined as determining the shape of the crack L given that the scattered field is known on $x_2 = l$. L is simulated as a perfectly conducting thin object, which is described in section 2.

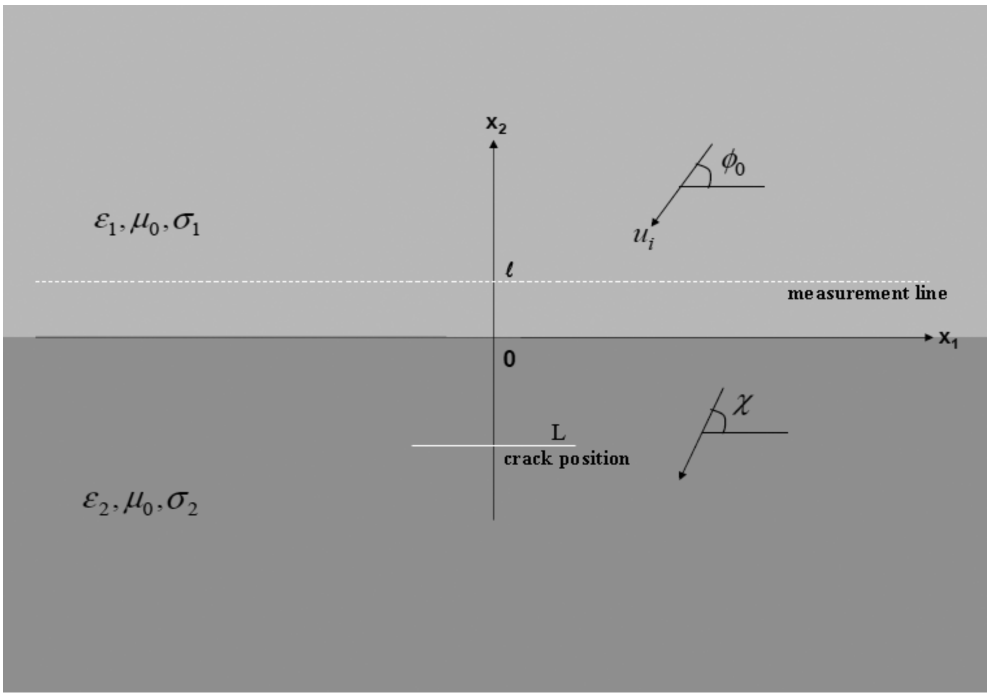

Figure 3.1: The geometry of the inverse scattering problem.

The crack is buried in the lower half-space, and it is illuminated by a plane wave u_i in the upper half-space as previously given in 2.1-2.2. The scattered field is measured on the finite line which is parallel to the planar interface.

3.2 An Analytical Continuation Method (ACM)

To solve the inverse crack problem described above an analytical continuation method will be presented. To this aim we will first reformulate the scattering problem by writing the scattered field in the form

$$u_s(\mathrm{x}) = \begin{cases} u_1(x), & x_2 > 0 \\ u_2(x), & x_2 < 0. \end{cases} \quad \textbf{(3.1)}$$

Then the scattering problem is rewritten as

$$\Delta u_1 + k_1^2 u_1 = 0 \quad , \mathrm{x}_2 > 0 \quad \textbf{(3.2)}$$

$$\Delta u_2 + k_1^2 u_2 = 0 \quad , \mathrm{x}_2 < 0 \quad \textbf{(3.3)}$$

$$u_1(x_1, 0) = u_2(x_1, 0) \quad \textbf{(3.4)}$$

$$\frac{\partial u_1(x_1,0)}{\partial x_2} = \frac{\partial u_2(x_1,0)}{\partial x_2} \quad \textbf{(3.5)}$$

and,

$$u_2(x) = -u_0(x) \ \ on \ \ L \quad \textbf{(3.6)}$$

with Sommerfeld radiation condition for infinity.

We consider the Fourier transform of $u_s(x)$ defined by

$$\hat{u}_s(\upsilon, x_2) = \int_{-\infty}^{\infty} u_s(x) e^{-i\upsilon x_1} dx_1 \quad \textbf{(3.7)}$$

Applications of the Fourier transform to the equations (3.2-3.6) yields

$$\frac{\partial^2 \hat{u}_1(\upsilon, x_2)}{\partial x_2^2} + (k_1^2 - \upsilon^2)\hat{u}_1(\upsilon, x_2) = 0 \quad , \mathrm{x}_2 > 0 \quad \textbf{(3.8)}$$

$$\frac{\partial^2 \hat{u}_2(\upsilon, x_2)}{\partial x_1^2} + (k_2^2 - \upsilon^2)\hat{u}_2(\upsilon, x_2) = 0 \quad , \mathrm{x}_2 < 0 \quad \textbf{(3.9)}$$

$$\hat{u}_1(\upsilon,0)=\hat{u}_2(\upsilon,0) \tag{3.10}$$

$$\frac{\partial\hat{u}_1(\upsilon,0)}{\partial x_2}=\frac{\partial\hat{u}_2(\upsilon,0)}{\partial x_2} \tag{3.11}$$

$$\hat{u}_2(\upsilon,x_2)=-\hat{u}_0(\upsilon,x_2) \quad , \mathrm{x}_2\in L \tag{3.12}$$

with $\hat{u}_1\to 0$ as $x_2\to\infty$, and $\hat{u}_2\to 0$ as $x_2\to-\infty$.

The solutions of the equations (3.8-3.11) are given by

$$\hat{u}_1(\upsilon,x_2)=A(\upsilon)e^{\gamma_1(\upsilon)x_2} \quad , \mathrm{x}_2>0 \tag{3.13}$$

$$\hat{u}_2(\upsilon,x_2)=B(\upsilon)e^{\gamma_2(\upsilon)x_2} \quad , \mathrm{x}_2<0 \tag{3.14}$$

Here $A(\upsilon)$ and $B(\upsilon)$ are unknown spectral coefficients to be determined while $\gamma_j(\upsilon)$ denotes the square root functions defined by

$$\gamma_j(\upsilon)=\begin{cases}\sqrt{\upsilon^2-k_j^2} & , |\upsilon|>k_j\\ -i\sqrt{k_j^2-\upsilon^2} & , |\upsilon|<k_j\end{cases} \tag{3.15}$$

for real k_j values.

In the inverse scattering problem considered here it is assumed that the scattered field is known on the line $x_2=l,\ l$>0. Let us denote this measured value by $u_s(x,l)$. Then its Fourier transform can be evaluated as

$$\hat{u}_s(\upsilon,l)=\int_{-\infty}^{\infty}u_s(x,l)e^{-i\upsilon x_1}dx_1 \tag{3.16}$$

By comparing this with equation 3.13, one can easily show that the unknown coefficient $A(\upsilon)$ can be given as

$$A(\upsilon)=\hat{u}_s(\upsilon,l)e^{\gamma_1(\upsilon)l}. \tag{3.17}$$

Thus, the measured field directly gives the unknown coefficients through the equations 3.17. The conditions 3.8-3.11 and 3.12 then yield to obtain $B(\upsilon)$, namely,

$$B(\upsilon)=\hat{u}_s(\upsilon,l)e^{\gamma_1(\upsilon)l} \tag{3.18}$$

Then, the scattered field in the region $x_2<0$ can now be written as

$$u_2(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{u}_s(\upsilon,l)e^{\gamma_1 l}e^{\gamma_2 x_2}e^{i\upsilon x_1}d\upsilon \quad , \mathrm{x}_2 < 0. \qquad \textbf{(3.19)}$$

The term $\hat{u}_s(\upsilon,l)e^{\gamma_1 l}e^{\gamma_2 x_2}$ is nothing but the analytical continuation of the measured field $u_s(x_1,l)$ to the region $x_2 < 0$ in terms of plane waves in the spectral domain. Of course, this analytical continuation is ill-posed. This ill-posedness is due to the fact that $\hat{u}_s(\upsilon,l)$ is multiplied by the factor $e^{\gamma_1 l}$ which means that the error in $\hat{u}_s(\upsilon,l)$ will be amplified with $e^{\gamma_1 l}$. This is what makes the analytical continuation ill-posed. Thus some regularization procedure has to be applied.

This is done by restricting the $(-\infty,+\infty)$ integration domain in the inverse Fourier transform to the interval $(-k_2,k_2)$), namely,

$$u_s(x) \simeq \frac{1}{2\pi}\int_{-k_2}^{k_2} \hat{u}_s(\upsilon,l)e^{\gamma_1 l}e^{\gamma_2 x_2}e^{i\upsilon x_1}d\upsilon \qquad \textbf{(3.20)}$$

The equation 3.20 is an approximation for the scattered field in the region $x_2 < 0$.

This approximate field will be used in the reconstruction procedure. More precisely, we choose a reconstruction domain, see Figure 3.2, in $x_2 < 0$ and search the points where $u_s(x) = -u_0(x)$.

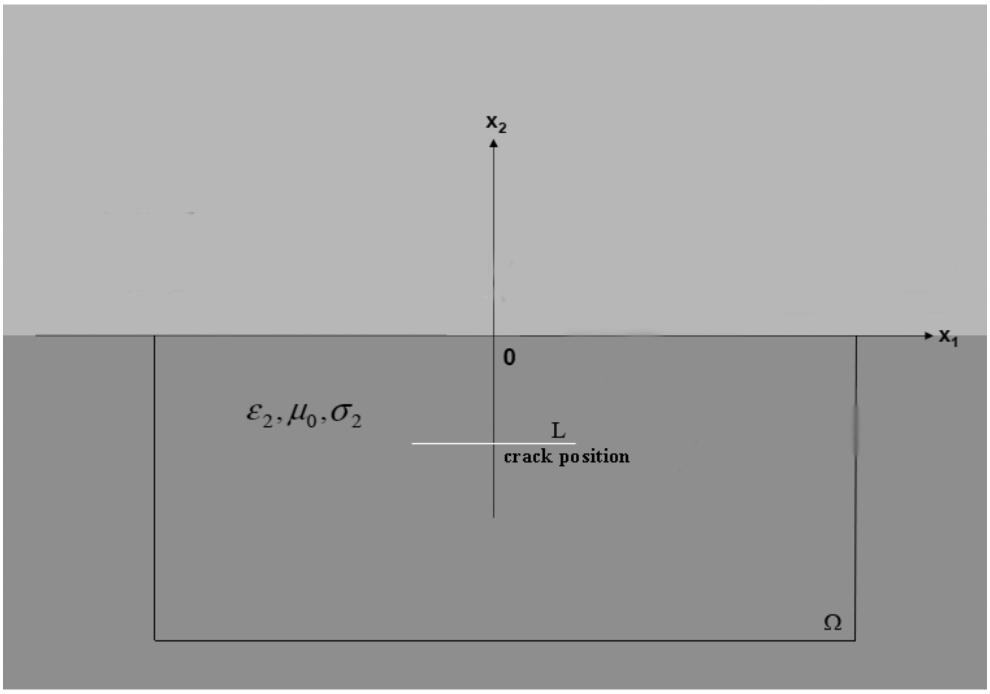

Figure 3.2: Representation of the reconstruction domain Ω.

4. SIMULATION RESULTS

In this part of the thesis numerical results for both the direct and the inverse scattering simulations are shown for comparison. In the first part of the study, the direct problem is solved by applying method of moments for two half-space media separated by planar interface, and then the crack buried in the lower half-space is simulated. The scattered field is then calculated and compared with simulation of the inverse problem which is achieved by applying an analytic continuation method (ACM). Finally the numerical results are shown for reconstruction of the crack in layered media using inverse scattering problem.

4.1 Numerical results for scattered field and crack reconstruction

In the Figures below, numerical results for total field are shown. Frequency chosen in the simulation is 300 MHz, and the angle of the incident plane wave is $\phi_i = \pi / 2$. Upper half-space is assumed to be free space and the lower half-space a dielectric medium. The scattered field is measured on the measurement line $l = 0.125m$, and measured data is used for the solution of the inverse scattering problem, reconstruction, namely. The scattered field is shown in the Figure 4.1.

In the first example, the lower half-space is chosen to be a lossless dielectric medium with a relative dielectric permittivity $\varepsilon_{r2} = 2$. A 1 m long and 0.02 m wide crack is located vertically in the lower half-space, having its center 1m below the planar surface.

Figure 4.2 and Figure 4.3 show the total field simulations of the direct and inverse problems, respectively. Using the scattered field data provided by the direct scattering problem solution shown in Figure 4.1, the inverse scattering simulation shows the crack clearly where the total field is equal to zero. It is shown in the Figure 4.3.

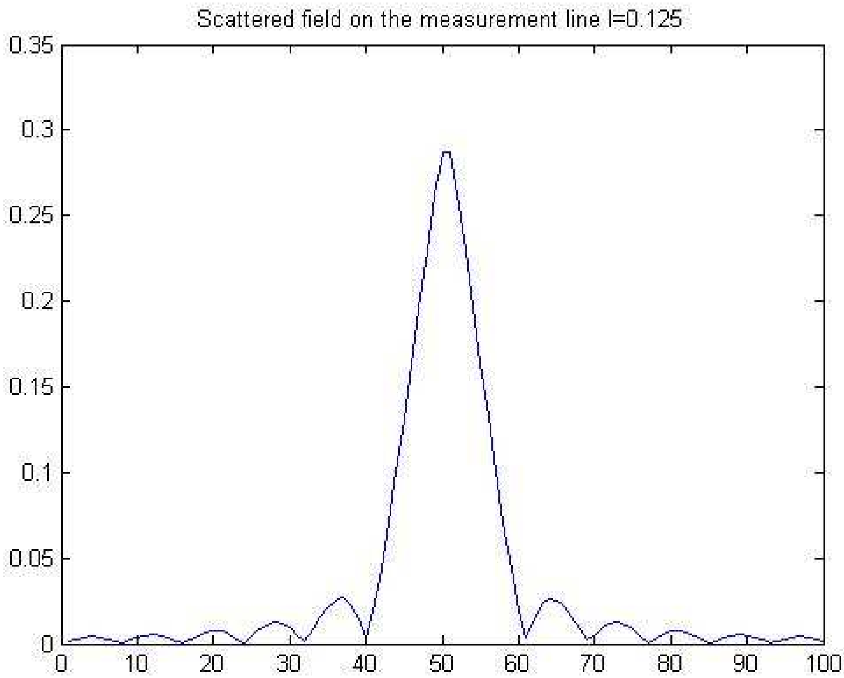

Figure 4.1 : Scattered field measured at the $l = 0.125m$.

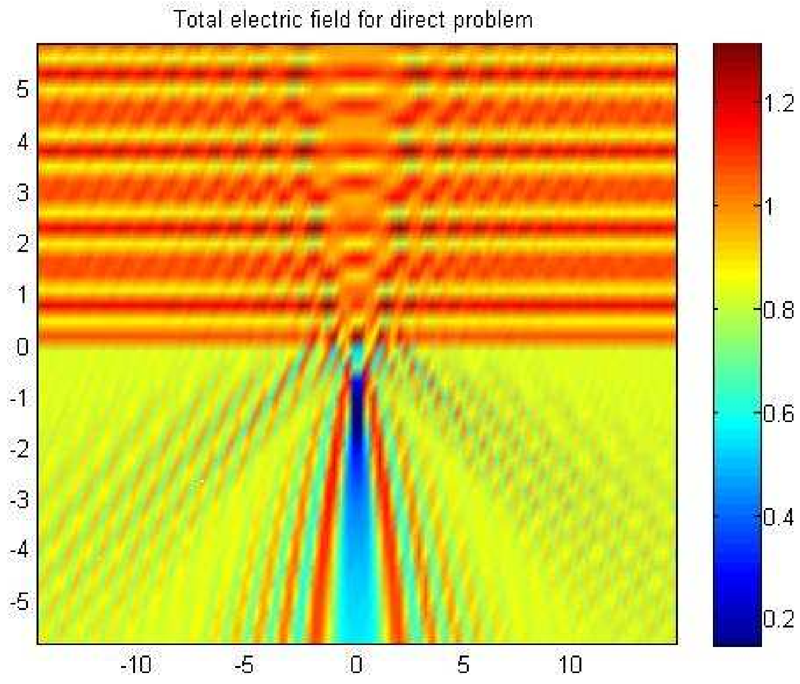

Figure 4.2 : Magnitude of the total electric field simulated by solution of the direct scattering problem.

In the second example, the lower half-space is chosen to be a lossy dielectric medium with a relative dielectric permittivity $\varepsilon_{r2} = 5$ and a conductivity $\sigma_2 = 10^{-6}$ S/m. A 1m long and 0.02m wide crack is vertically located in the lower half-space, having its center 1m below the planar surface.

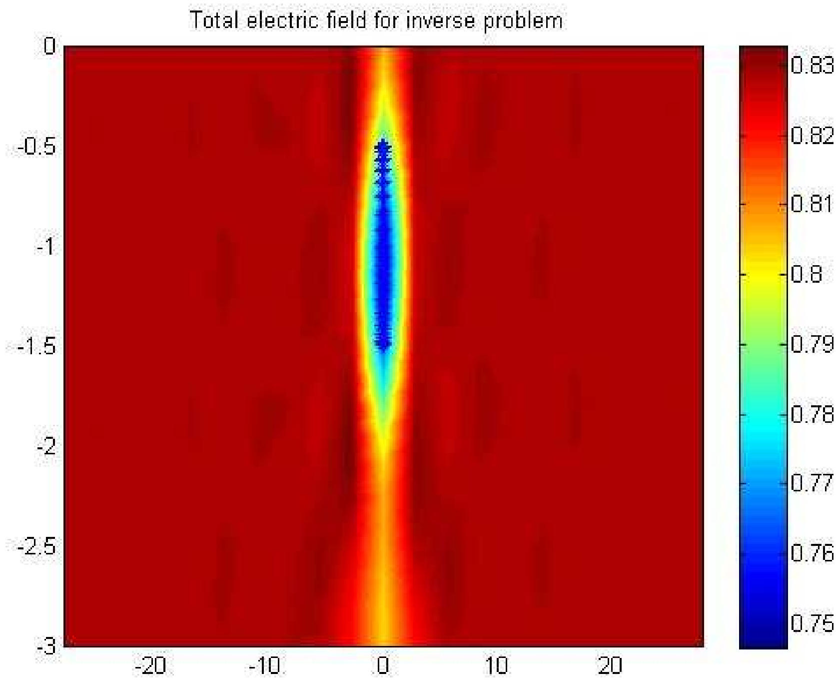

Figure 4.3 : Magnitude of the total electric field simulated by using analytical continuation method (ACM)

In the Figure 4.4 scattered field achieved by the solution of the direct scattering problem is shown. Comparing Figure 4.4 with previously shown Figure 4.1 it is obvious that the scattered field has decreased in amplitude almost twice, what is expected due to the conductivity changes of the lower half-space.

Figure 4.5 and Figure 4.6 show the total field simulations of the direct and inverse problems, respectively. Since the contrast between the two media, the two half-spaces namely, is higher compared to the previous example, as it can be seen in Figure 4.5, the reflection on the planar interface is higher. In addition to the reflection due to the contrast, the conductivity in the lower half-space causes attenuation, leading to reduction in the scattered data. However, Figure 4.6 shows that the crack is still highly distinctive where the total field equals to zero.

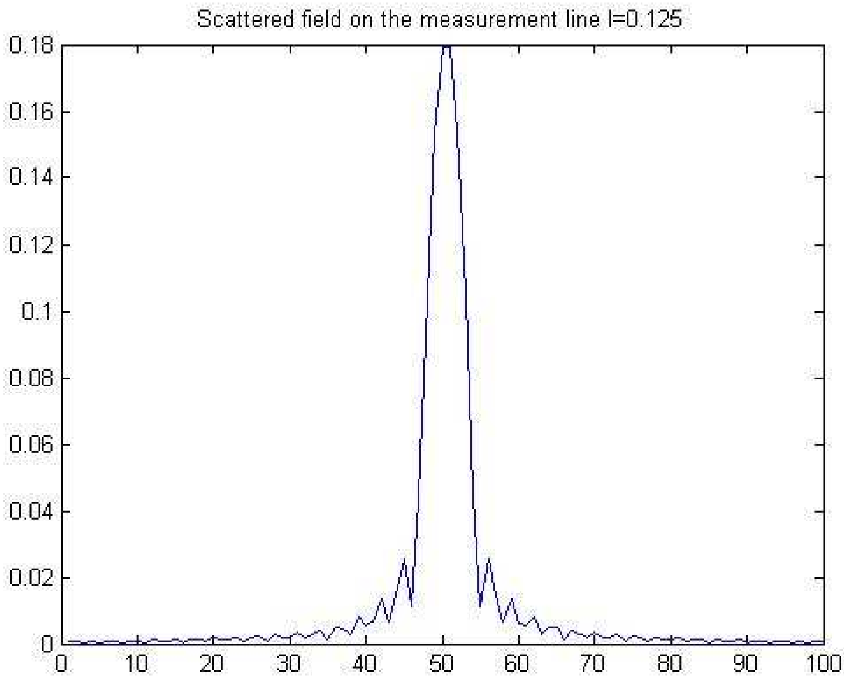

Figure 4.4 : Scattered field measured at the $l = 0.125m$.

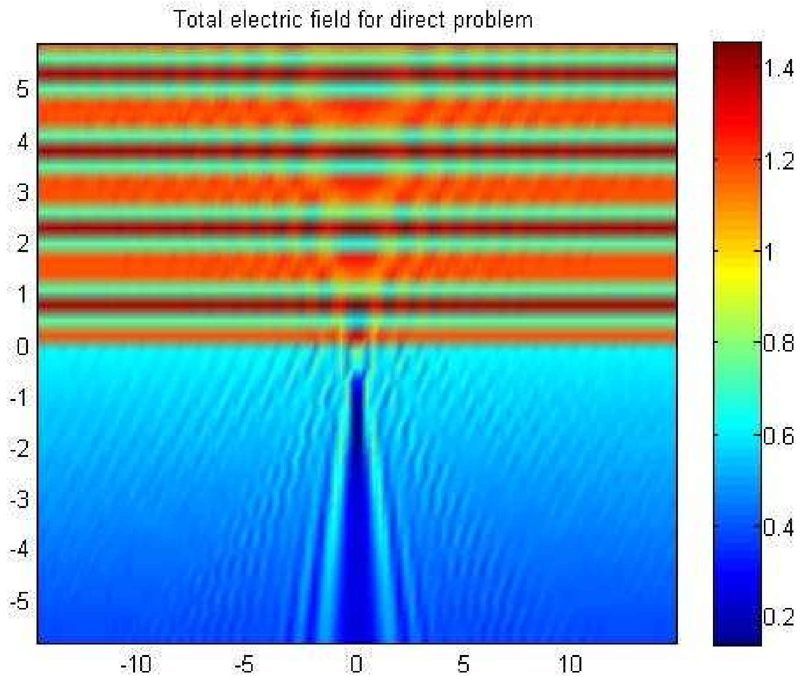

Figure 4.5 : Magnitude of the total electric field simulated by solution of the direct scattering problem.

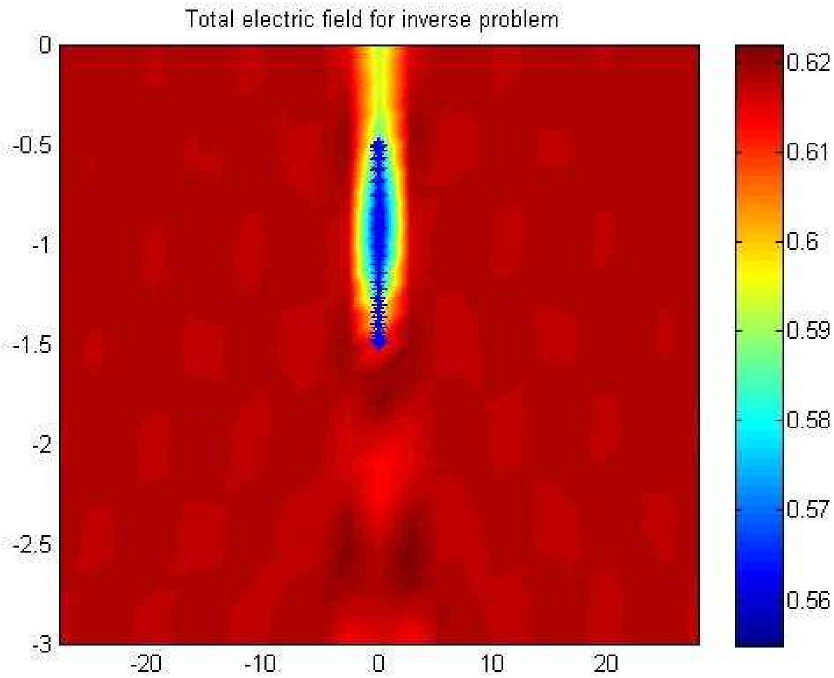

Figure 4.6 : Magnitude of the total electric field simulated by using analytical continuation method (ACM)

In the third and fourth examples, the lower half-space is chosen to be a lossless dielectric medium with a relative dielectric permittivity $\varepsilon_{r2} = 2$. The cracks used in these examples are now horizontally positioned at 1m and 2m respectively below the planar surface.

In the Figure 4.7 scattered field data is shown, comparing it with data gained from vertically positioned crack it can be seen that scattered field magnitude is greatly improved what is expected result.

As it can be seen in Figure 4.8, which shows the total electric field simulated using the direct scattering problem solution, the crack causes a total reflection of the electromagnetic waves, due to its perfect conductivity. Since the incoming wave is reflected totally, a shadow, where no data is obtained from, appears behind the crack in the direct problem. The scattered field, which lacks the information about what's behind the crack, is used to simulate the inverse problem, whose total field can be seen in Figure 4.9. Although the location of the crack can be estimated weakly in Figure 4.8, as it is expected, its shape is not possible to recognize easily.

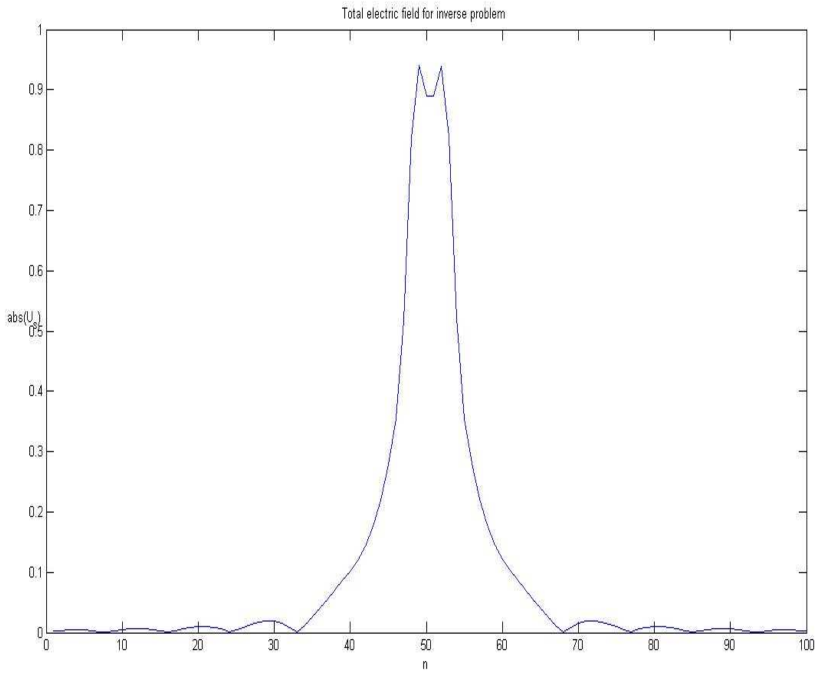

Figure 4.7: Scattered field measured at the $l = 0.125m$.

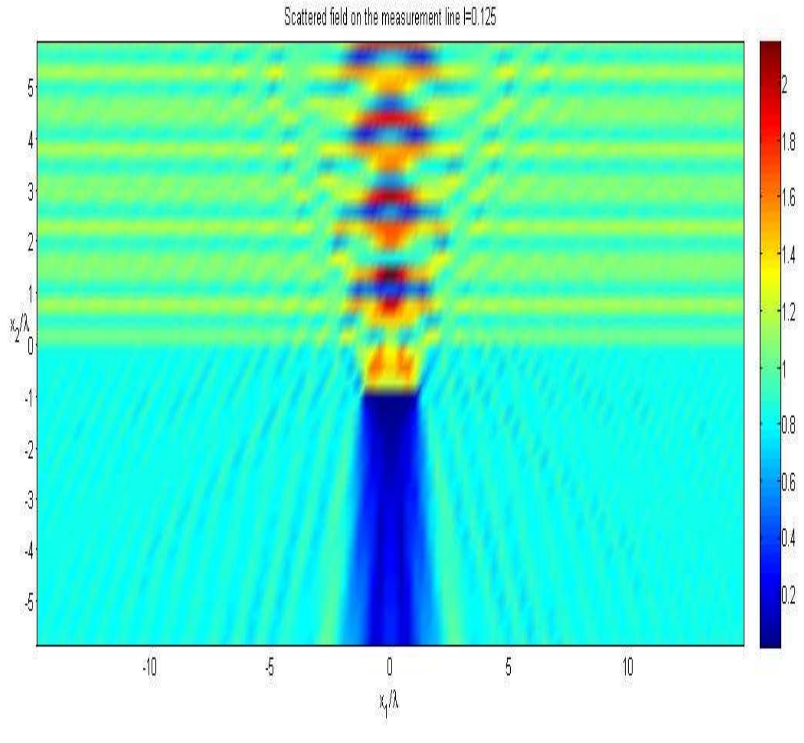

Figure 4.8 : Magnitude of the total electric field simulated by solution of the direct scattering problem, vertically positioned crack case.

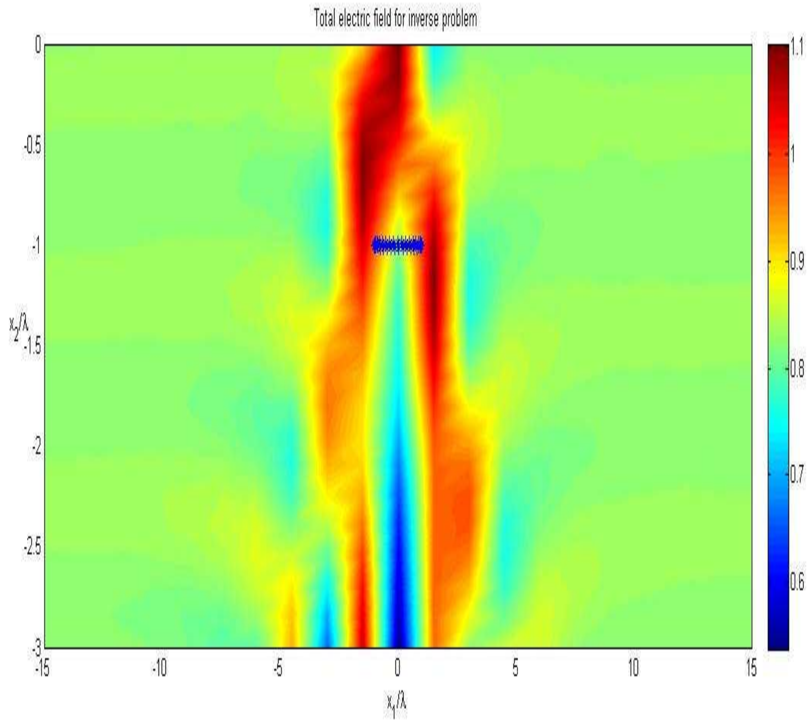

Figure 4.9 : Magnitude of the total electric field simulated by using analytical continuation method (ACM).

For the case when crack's length is kept at the same value of 2λ and thickness is also kept as 0.04λ, but depth is extended to $x_2 = -2\lambda$ scattered field as well as the magnitude of the total electric field obtained from solution of the direct problem is shown in Figure 4.10 and 4.11 respectively.

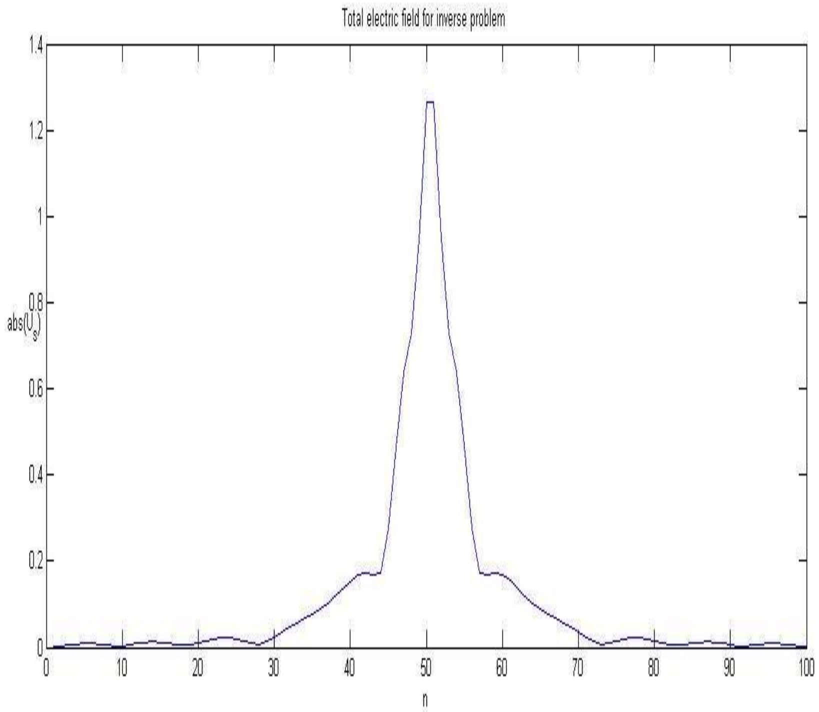

Figure 4.10 : Scattered field measured at the $l = 0.125m$.

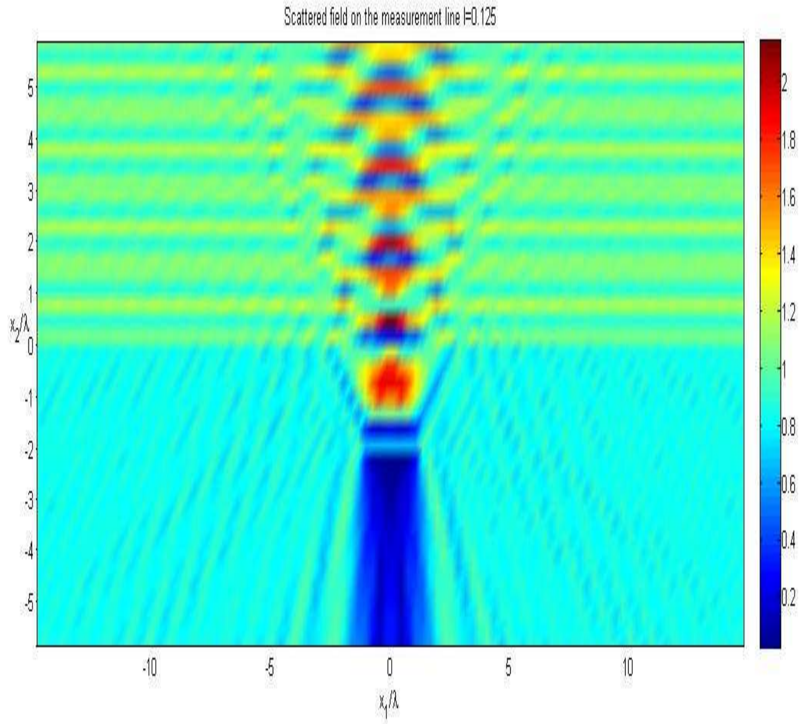

Figure 4.11 : Magnitude of the total electric field simulated by solution of the direct scattering problem, vertically positioned crack case.

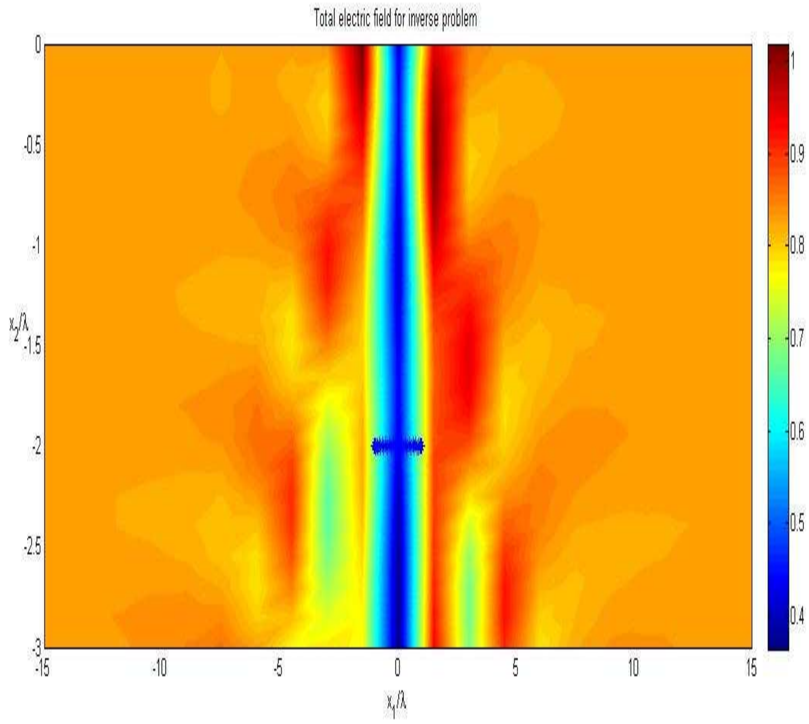

Figure 4.12 : Magnitude of the total electric field simulated by using analytical continuation method (ACM).

From the results in above given Figures 4.7-4.9 and 4.10-4.12 respectively, we can conclude that depth of the buried crack as well as length of the observation line x_1 have significant impact on reconstruction results. Observation line's length influence on the reconstruction results are shown in the Figures 4.13-4.15.

As it can be seen in Figure 4.13, which shows the total electric field simulated using the direct scattering problem solution, the crack causes a total reflection of the electromagnetic waves, due to its perfect conductivity. Since the incoming wave is reflected totally, a shadow, where no data is obtained from, appears behind the crack in the direct problem. The scattered field, which lacks the information about what's behind the crack, is used to simulate the inverse problem, whose total field can be seen in Figure 4.14. Although the location of the crack can be estimated weakly in Figure 4.15, as it is expected, its shape is not possible to recognize easily.

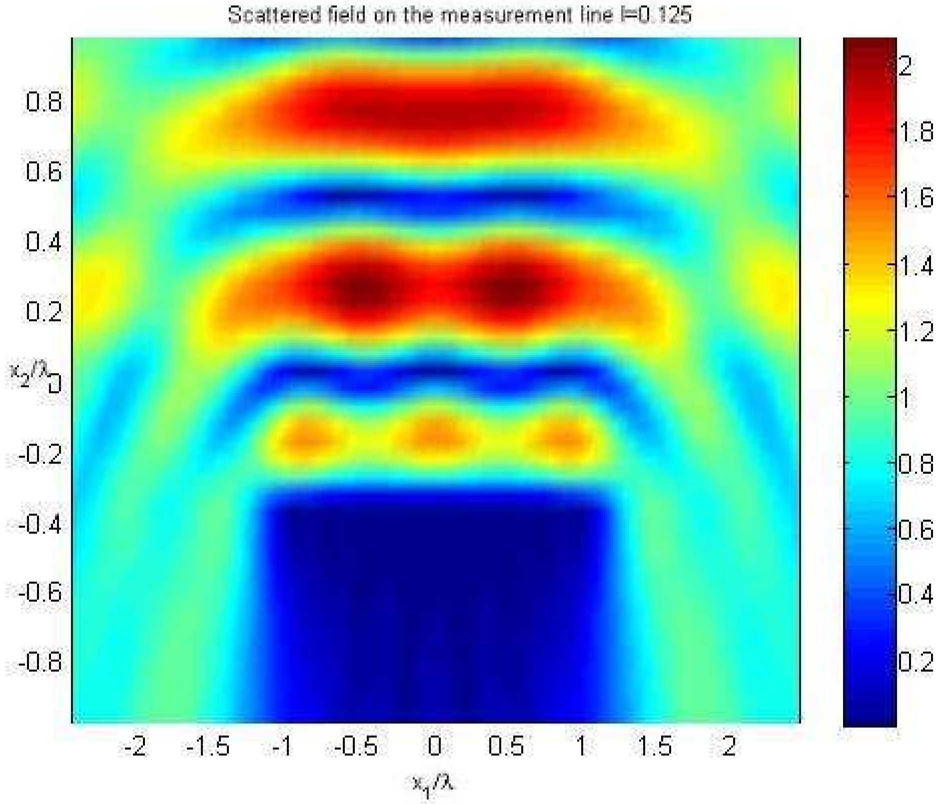

Figure 4.13 : Magnitude of the total electric field simulated by using MoM.

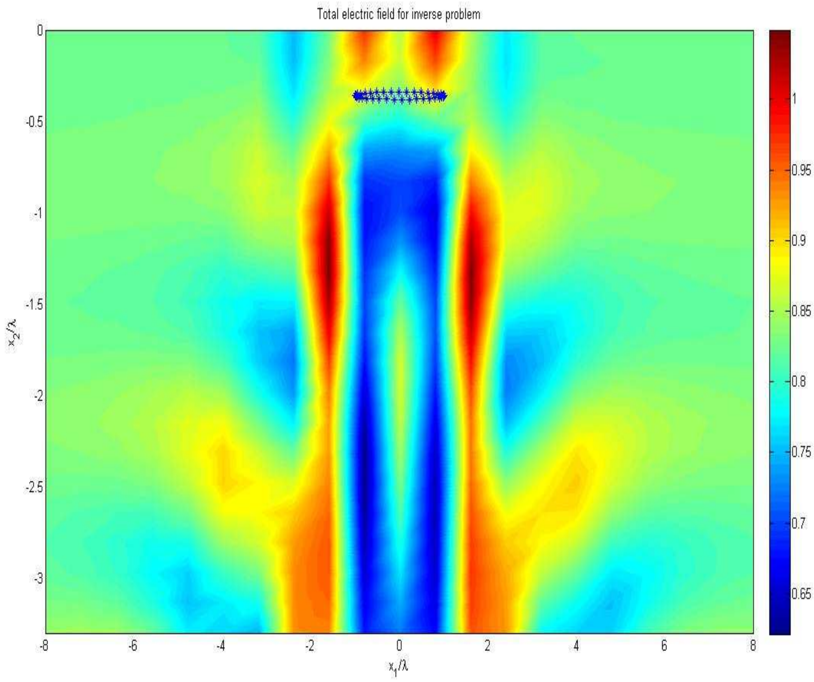

Figure 4.14 : Magnitude of the total electric field simulated by using analytical continuation method (ACM).

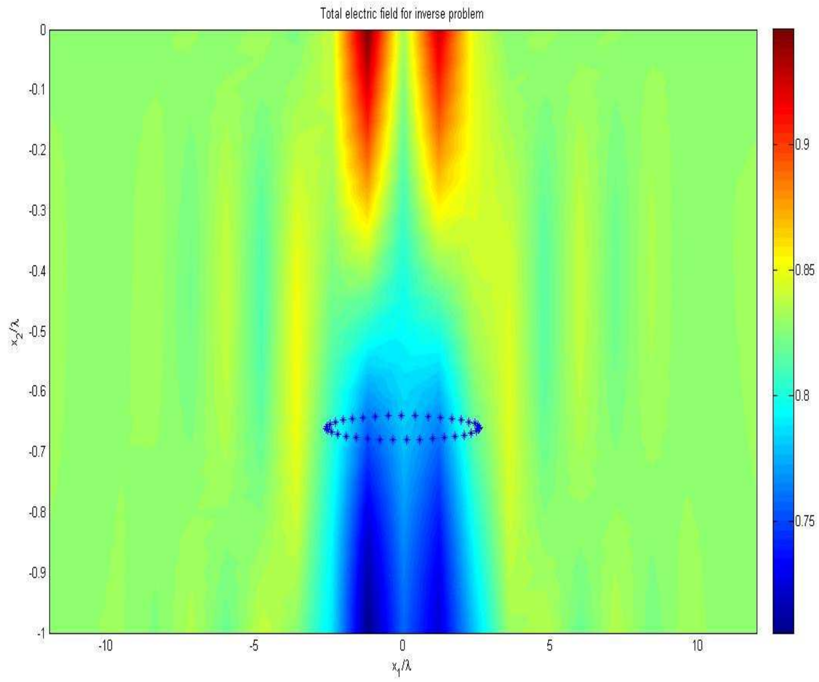

Figure 4.15 : Magnitude of the total electric field simulated by using analytical continuation method (ACM).

Figures above justifies crack depth's and length's influence on reconstruction results. In this thesis Intel Core 2 Duo P8600 processor has been used with 4GB of DDR3 memory. Average simulation time is 200 seconds.

5. CONCLUSION

In this thesis both direct and inverse scattering problems related to cracks buried in a half-space are investigated. A MoM solution is presented for the direct scattering problem by ignoring the end contributions of crack in terms of 2D objects of small thickness. On the other hand an analytical continuation method, based on plane wave spectral representation of the scattered field, is given for the solution of inverse crack problem.

The MoM approach yields accurate results for the direct problem, while the analytical continuation gives good approximation for the scattered field in the lower half space in the case when crack is located vertically. Poor reconstructions are obtained for horizontally positioned cracks. Thus the method is more useful for reconstruction of the cracks in the structures where the crack is vertically positioned.

In this thesis, main aim was to discuss on cracks and their behaviour in two-half space media. Also formulation for an analytical continuation method was derived and results of that method are compared with results of direct scattering problem. Two half-space medium is assumed to be with planar interface, with different media k_1 and k_2 respectively. Scattered field data was determined for upper half-space firstly, and then we derived, on the basis of that scattered field, scattered field on the two-half space planar interface. Due to field on the planar interface we derived field in the lower half-space. Final step was to reconstruct buried crack from the lower half-space, using numerical methods and applying continuation method. Also some future work on this subject in the case of rough surface can be area of interest.

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CURRICULUM VITAE

Candidate's full name: Kemal MRKONJA

Place and date of birth: Travnik, Bosnia and Herzegovina, 1980

Permanent Address: Küçükbahçe Sok. Emek Apt. No: 30/14 Osmanbey, Şişli, Istanbul, Turkey.

Universities and Colleges attended:

2007- : M.Sc in Telecommunication Engineering, Istanbul Technical University, Turkey

2001-2006 : B.Sc in Electronics and Telecommunication Engineering, Yildiz Technical University, Turkey.